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BILOCATION AND BIVALUE GRAPHS

I am indebted to David Eppstein, a computer science professor at the University of California, Irvine, for allowing me to spread the word on the techniques in this chapter, which he developed.

BILOCATION GRAPHS

In a bilocation graph, you make a diagram of cells that contain candidates that appear in no other possible places in that particular row, column, or box.

NONREPETITIVE BILOCATION CYCLE

To find a nonrepetitive bilocation cycle you start at any cell, then move to another cell in the same row, column, or box so that the cell you are moving from and the cell that you are moving to are the only two cells in that particular row, column, or box that can have a particular number. Then from the new cell, pick a different number, and find a new cell that is in its row, column, or box (making sure that there are no other places for that new number in that particular row, column, or box). Continue like this until you get back to the starting cell, and you've found a nonrepetitive bilocation cycle. Here's an example and the sticking point:

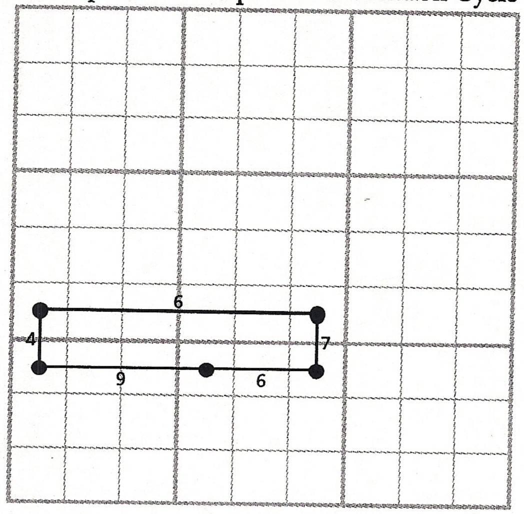
Example 32

			7		1			4
1		4	2			5		3
				3		6		
		8			9	7	6	
	9	5	1			8		
		1		2				
3		6			5	4		9
8			4		3			

Example 32-1

56	3568	39	7	569	1	2	89	4
1	67	4	2	69	8	5	79	3
579	58	2	59	3	4	6	1	78
2	134	8	35	45	9	7	6	15
67	136	37	356	8	2	9	4	15
46	9	5	1	47	67	8	3	2
4579	45	1	69	2	67	3	58	78
3	27	6	8	1	5	4	27	9
8	25	79	4	79	3	1	25	6

Example 32 Nonrepetitive Bilocation Cycle



This diagram shows cells 61 and 66 connected by a 6, since a 6 has to appear in one of those two cells; cells 66 and 76 connected by a 7, since a 7 has to appear in one of those two cells; cells 76 and 74 connected by a 6, since a 6 has to appear in one of those two cells; cells 74 and 71 connected by a 9, since a 9 has to appear in one of those two cells; and cells 71 and 61 connected by a 4, since a 4 has to appear in one of those two cells. It makes a complete cycle, and the numbers along the edges don't repeat along any two sections that share a node.

So we know that in the cells with the five node points, the numbers starting at the upper left and going around clockwise will either be 4-6-7-6-9 or 6-7-6-9-4. What this means is that cell 71 has to be either a 4 or a 9, so we can remove the 5 and 7 from its candidate list. That means that the 5 in column 1 must appear in either cell 11 or cell 31, which in turn means, via interaction, that cell 32 cannot be a 5. Removing the 5 candidate from cell 32 is the magic moment. It forces that cell to be an 8, and lets us coast to the finish line.

Example 32 Answer

6	3	9	7	5	1	2	8	4
1	7	4	2	6	8	5	9	3
5	8	2	9	3	4	6	1	7
2	1	8	3	4	9	7	6	5
7	6	3	5	8	2	9	4	1
4	9	5	1	7	6	8	3	2
9	4	1	6	2	7	3	5	8
3	2	6	8	1	5	4	7	9
8	5	7	4	9	3	1	2	6

REPETITIVE BILOCATION CYCLE

A repetitive bilocation cycle is the same thing, except that there is exactly one case where the numbers coming out of two consecutive nodes in the cycle are the same.

Example 33

8				1			6	9
	4				3			
	6	1	5			8		
3	1	6		4				
				9		1	7	3
		7			6	4	1	
			3				8	
6	8			7				2

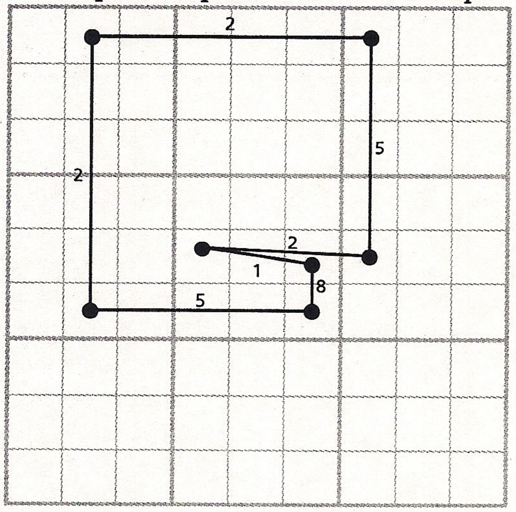
Eventually you get to this grid:

Example 33-1

8	25	3	47	1	47	25	6	9
59	4	29	8	6	3	7	25	1
7	6	1	5	2	9	8	3	4
3	1	6	27	4	57	9	25	8
59	7	89	12	3	158	25	4	6
4	25	28	6	9	58	1	7	3
2	3	7	9	8	6	4	1	5
1	9	4	3	5	2	6	8	7
6	8	5	14	7	14	3	9	2

Here is one bilocation graph you can form:

Example 33 Repetitive Bilocation Graph



The graph shows a complete cycle. Between any two nodes, the number on the line must go in one of the two nodes. So, for example, the 8 must go in either cell 56 or 66. Now examine the upper left node. It has a 2 coming out of both sides. If cell 12 were not a 2, that would force cells 17 and 62 to be 2's, which wouldn't leave enough room for all the other numbers in the cycle to fit in. The 5's would be forced into cells 57 and 66, which would force a 2 in cell 54 and an 8 in cell 56, leaving no place for the 1 that is limited to cells 54 and 56. So cell 12 must be a 2. From there it's a cakewalk.

Example 33 Answer

8	2	3	7	1	4	5	6	9
5	4	9	8	6	3	7	2	1
7	6	1	5	2	9	8	3	4
3	1	6	2	4	7	9	5	8
9	7	8	1	3	5	2	4	6
4	5	2	6	9	8	1	7	3
2	3	7	9	8	6	4	1	5
1	9	4	3	5	2	6	8	7
6	8	5	4	7	1	3	9	2

BIVALUE GRAPHS

Bivalue graphs are much easier to find. They are cycles of buddy cells that each have just two candidates in them. They must be nonrepetitive to be helpful. Here's an example:

Example 34

5							3	
	9	8		6			7	
				2	3	1		
	1		9					
		7				3		
					5		8	
		2	4	9				
	7			5		4	6	
	6							2

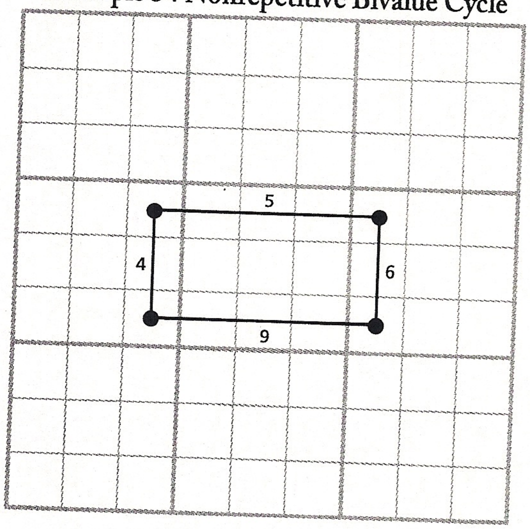
We get this far using the standard methods:

Example 34-1

5	2	1	78	478	9	68	3	468
3	9	8	15	6	14	2	7	45
7	4	6	58	2	3	1	59	589
2468	1	45	9	3	2478	56	24	67
24689	58	7	1268	148	1248	3	24	1569
2469	3	49	1267	147	5	69	8	1679
18	58	2	4	9	6	7	15	3
19	7	39	1238	5	128	4	6	89
149	6	3459	1378	178	178	589	19	2

Cells 43, 47, 67, and 63 form a nonrepetitive bivalue graph, as shown below.

Example 34 Nonrepetitive Bivalue Cycle



In the cyclic sequence of cells 43, 47, 67, 63, and back to 43, each cell has two candidates, each of which can also be placed at one of the cell's two neighbors in the sequence. Starting in the upper left of the cycle and going around clockwise, the cells will be either 5-6-9-4 or 4-5-6-9. In either case, a 4 must go in either 43 or 63, so we can eliminate the 4 candidates from cells 41, 51, 61, and 93. Also, a 6 must go in either 47 or 67, allowing us to eliminate the 6 candidates from 49, 59, 69, and 17. Finally, a 9 must go in either 63 or 67, so we can eliminate the 9 candidate from 61 and 69. The most helpful of these results is the elimination of the 6 candidate from cell 17, which forces it to be an 8, allowing us to finish off the puzzle posthaste.

Example 34 Answer

5	2	1	7	4	9	8	3	6
3	9	8	5	6	1	2	7	4
7	4	6	8	2	3	1	5	9
2	1	5	9	3	8	6	4	7
9	8	7	6	1	4	3	2	5
6	3	4	2	7	5	9	8	1
8	5	2	4	9	6	7	1	3
1	7	9	3	5	2	4	6	8
4	6	3	1	8	7	5	9	2

Lots more information about these methods can be found in the paper "Nonrepetitive Paths and Cycles in Graphs With Application to Sudoku" by Professor Eppstein, on the Web at arxiv.org/abs/cs.DS/0507053.

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GUESSING

You now know all that you need to know to solve all the puzzles in the puzzle section of this book. What do you do if, when solving puzzles from another source, you get to a point where none of the methods in this book can finish off the puzzle? My first recommendation would be to no longer solve puzzles from that source. Any sudoku author who requires you to guess is not making quality puzzles. If even single-iteration chains and grid colorings result in nothing definitive, the only thing left is bifurcation, or flat-out guessing, where you simply choose a candidate for a cell as if it were the right one and solve from that point to see where it leads. If it's the incorrect choice it will ultimately lead to some kind of contradiction, and you'll then go back to your bifurcation point and choose a different candidate and go through the process again. You'll want to choose as a bifurcation point a cell with as few candidates as possible, preferably two. However, it's possible that even this will not be enough to solve the puzzle, as it may require multilayered bifurcation in order to crack it. This guess-and-check method is also known by its Japanese name, "nishio."

It's also very helpful to guess correctly, since then you'll stumble onto the answer and not have to go back to try the other possibilities. Peeking at the answer grid for the contents of the key cell is an *excellent* way to keep your guessing batting average close to perfect (just make sure nobody sees you doing it).

Here is a puzzle, and the point where we reach a screeching halt:

Example 35

		2			8	3		
			4	7				6
4			2				1	
		4					9	
1		6				4		7
	9					2		
	2				7			3
7				1	4			
		3	9			7		

Example 35-1

569	156	2	156	569	8	3	7	4
3569	1358	589	4	7	1359	58	2	6
4	35678	578	2	356	356	58	1	9
2	3578	4	5678	3568	356	1	9	58
1	58	6	58	29	29	4	3	7
358	9	578	1578	4	135	2	6	58
5689	2	1	568	568	7	69	4	3
7	568	589	3	1	4	69	58	2
568	4	3	9	2568	256	7	58	1

The candidates at cell 77 are 6 and 9. If you choose a 6, it almost works, and you can get down to having just two uncommitted cells left (cells 26 and 35), but it can't be finished.

If you choose a 9 instead, you can solve it all the way to the end. You of course noticed the Gordonian Rectangle at cells 23, 33, 27, and 37 that allowed you to put a 7 in cell 33, right?

Example 35 Answer

5	6	2	1	9	8	3	7	4
9	1	8	4	7	3	5	2	6
4	3	7	2	6	5	8	1	9
2	7	4	8	3	6	1	9	5
1	8	6	5	2	9	4	3	7
3	9	5	7	4	1	2	6	8
8	2	1	6	5	7	9	4	3
7	5	9	3	1	4	6	8	2
6	4	3	9	8	2	7	5	1

No other puzzles in this book require you to guess like this.

TO GUESS OR NOT TO GUESS

Back on page 18 I wrote, "Should you guess? The answer to that is an emphatic no." So why am I saying you should guess here? The simple answer is that I lied on page 18. Please forgive me, but there was a reason. The only time you'll need to guess is when you are doing a ridiculously hard puzzle written by someone who thinks guessing should be allowed. Good sudoku writers don't allow that, so if you're doing puzzles from a good source (anything by Frank Longo is sure to be good), then you'll never have to guess.

Also, if I had said it was okay to guess, you would have been guessing left and right instead of learning all the strategies you need to avoid guessing. So now that you know everything, try all the other strategies, and if they all fail, then I still emphatically say you shouldn't guess. You should instead throw away the puzzle and get a new puzzle from a source that doesn't require guessing. If you can't stand the idea of throwing away an uncompleted puzzle, then be sure to switch pencil colors so you know what to erase if you come to a dead end (see the sidebar on page 44 for more on that).